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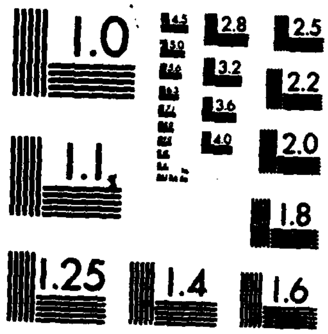


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# HYBRID EXPERIMENTAL-NUMERICAL STRESS ANALYSIS

by

A. S. Kobayashi

April 1983



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# HYBRID EXPERIMENTAL-NUMERICAL STRESS ANALYSIS

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## ABSTRACT

The hybrid experimental-numerical stress analysis technique, which saw limited applications during the 1950's, has been resurrected with the vastly improved numerical techniques of the 1970's. By inputing the experimental results as initial and boundary conditions, modern computer codes can be executed in its generation and application modes to yield results which are unobtainable when only one of the two techniques is used. The hybrid technique thus exemplifies the complementary role of the experimental and numerical techniques.

## INTRODUCTION

One of the frustrations of an experimental stress analyst is the lack of a universal experimental procedure which solves all problems. Referred to as his second principle, Durelli states that "Seldom does one method give a complete solution, with the most efficiency [1]". Examples of this second principle is seen in photoelastic coating and brittle coating techniques which require additional strain gage testing in locations of high stress concentrations identified by these two techniques. The hybrid experimental-numerical stress analysis technique is an aberration of the above where numerical stress analysis replaces the second experimental method.

Early applications of the hybrid experimental-numerical stress analysis technique were limited to separations of two- and three-dimensional states of

stress in photoelastic specimens. The first stress invariant obtained through a finite difference solution to the compatability equation and the maximum shear stress distribution provided by the isochromatics yielded the two planar components of the principal stresses [2,3,4]. The shear-difference method [2,5] and the Filon's method [6,7] used the isochromatic and isoclinic data to integrate the equilibrium equations along a straight line and a stress trajectory, respectively. These single-purpose numerical techniques thus provided only the stresses along a specified integration path.

In contrast to the above, the modern super codes based on finite element method, boundary element method and finite difference method yield the complete states of stress, strain and displacement for the given constitutive relations and boundary and initial conditions. The uncertainty or the lack of knowledge in these given conditions, however, limited the accuracy of the otherwise voluminuous outputs of these super codes. Inaccurate numerical modeling procedures generated results with obvious errors and are credited for the resurgence of three dimensional photoelasticity in the 1970's. The hybrid experimental-numerical stress analysis technique of today reduces, if not eliminates, the above uncertainties in prescribed input conditions by using experimentally determined boundary and initial conditions. The output from the otherwise proven numerical techniques are either the constitutive relation or the complete states of displacement, strain and stress which cannot be readily extracted through the use of a single experimental technique in stress analysis. Thus, the hybrid experimental-numerical technique is an extremely efficient stress analysis technique which often provides more information than needed. The full potential of the hybrid technique, however, is yet to be exploited because of the historic dichotomy between the theoreticians-turned numerical analysts and the experimentalists.

In the following, the utility of the hybrid experimental-numerical stress analysis technique will be demonstrated by some stress analysis problems involving two- and three-dimensional structural components, biomechanics and fracture mechanics.

## ELASTIC ANALYSIS OF STRUCTURAL COMPONENTS

The numerical techniques used in modern hybrid technique for structural analysis are vastly superior to their predecessors since they provide the entire states of stress, strain and displacements. As a straight forward extension of the classical hybrid technique, Rao [8] used measured temperature and surface traction data to solve, by the finite difference method, the Beltrami-Michell stress equations of compatiblity interior to an axisymmetric solid. Figure 1a shows the end retaining ring, which is shrink-fitted to the two ends of the vector and which is used to contain the end loops of rotor windings, in a turbo-generator. The distributions of hoop stress, which is generated by shrink fitting and the centrifugal force, obtained by the hybrid technique, three dimensional frozen stress photoelasticity and a two-dimensional analog are shown in Figure 1b. The utility of this hybrid technique is demonstrated by the author's quote of "The time-needed for the analysis is smaller than that required by the time-consuming and tedious shear-difference methods" [8].

Figure 2 shows a water turbine wheel and its curvilinear finite difference grid representation which was analyzed by Barishpolsky [9]. Frozen stress photoelasticity was used to determine the stress tensor on the complex boundaries. These boundary values were input to the curvilinear finite difference equations for three-dimensional elasticity where the number of equations equalled the number of nodes and thus reduced the computational time by

three to six folds over standard finite difference codes. The procedure is extended to steady state, three-dimensional problems where measured surface temperature must be input in addition to the measured surface tractions [10].

While the specialized codes used in the above hybrid techniques are computationally more efficient by design, off-shelf codes in finite element and boundary element methods are often used for sheer expediencies. For an elasto-static problem, the boundary element method is more computationally efficient and natural where the input data consists of experimentally determined boundary displacements and tractions. When used together with the double exposure, laser speckle interferometry, "the measured surface displacements become the input data needed in the boundary element method to calculate the traction vectors at specified points on the boundary [11]" as well as in the interior of the body. Moslehy and Ranson [11] demonstrated the utility of this hybrid technique by the excellant agreements in theoretically and experimentally obtained stresses interior to a cantilever beam with a transverse end load. In a similar application of the hybrid technique, Balas, Sladek and Drzik [12] used the double-aperture, laser speckle interferometry and demonstrated the advantage of the hybrid technique by analyzing only the region of interest of a plate-stiffened frame. In this case, the recorded displacements were input to a simplified boundary, which is represented by the dashed lines, of the frame structure shown in Figure 3. Boundary element method was used to determine the stress distributions along the three cross sections shown in Figure 4.

As a variation in the above mentioned hybrid technique, Umeagukwu, Peters and Ranson [13] used two-dimensional photoelasticity together with a boundary element code to optimize the filets in a doubled notch tension plate. The interior principal stresses obtained by the hybrid technique were used to

more evenly distribute the load along the net section and thus resulted in a better understanding of the filet optimization problem.

## CORNEO-SCLERAL ENVELOPE

The interocular pressure of a human eye is maintained at a nearly constant level of 15 - 20 mmHg through a complex physiological system involving the mechanical, biochemical and neurological responses of the eye [14]. When the outflow of the ocular fluid is restricted by pathological conditions, the ensuing increase in interocular pressure eventually results in glaucoma which is the direct cause of 13.5 % of the blindness in United States [15]. Tonometry monitors this interocular pressure by measuring the exterior mechanical response of the cornea which is indented or flattened by a tonometer plunger. The tonometer reading is thus affected by the mechanical response of the pressurized corneo-scleral envelope which is essentially a pressure vessel containing the optical and neurological components.

The mechanical properties of the cornea and sclera are difficult to obtain because of the small size, delicacy and natural curvature. The commonly used ocular rigidity [16], which relates the pressure and volume of the corneo-scleral envelope, is a global coefficient and is not suitable for analyzing the local deformation process under tonometer loading. Simple tension testings of excised strips of the cornea [17] and the sclera [18] yielded erroneous modulus of elasticity and Poisson's ratio by the loosening of the collagen fibrils from the soft mucopolysaccharide at the excised edges. In order to overcome the deficiencies of the above global and local approaches, Woo et al [19,20] developed a hybrid experimental-numerical procedure for determining the local mechanical property of an intact corneo-scleral envelope.

Woo's experimental procedure consisted of measuring the cornea and sclera deformations as well as the volume changes of pressurized anterior segments of enucleated human eyes. A flying spot scanner was used to measure the relative motions of two white targets on the cornea or sclera which were mounted on a McEwen-type chamber [21]. Woo's numerical procedure consisted of matching, through trial and error, the measured and computed deformations and volume changes. A pressurized axisymmetric finite element model of the anterior segment of the corneo-scleral envelope was used to execute the finite element code in its application mode for this purpose. The resultant isotropic, trilinear, elastic stress-strain relations obtained for this analog model of the corneo-scleral envelope is shown in Figure 5. These trilinear stress strain relations were incorporated into a finite element of the total eye which was used to calculate the nonlinear intraocular pressure-volume relation. The lack of bending rigidity in the cornea under the tonometer probe was modeled by artificially reducing the bending stiffness of the finite elements in the compression region. With this modification, excellent correlations between the calculated and published experimental results were obtained [20].

The membrane shell elements, which were later used to construct the corneo-scleral envelope, shown in Figure 6 [22], removed the above mentioned artificial reduction in bending rigidity in the solid elements used by Woo. Woo's experimental data [19] was re-evaluated by this membrane shell model which yielded slightly different distribution of elastic moduli along the corneo-scleral shell. Such differences demonstrates the inevitable interdependence of the experimental data and numerical modeling of the hybrid experimental-numerical technique where the finite element model is used as an analog model of the experiment [23].

The anterior portion of the membrane finite element model was then used to model the deformation process under tonometer loading. Figure 7 shows the computed and measured [24] relations of probe force versus probe area under applanation tonometry.

ELASTIC-PLASTIC FRACTURE MECHANICS

Fracture parameters governing elastic, elastic-plastic and dynamic fracture, with the exception of geometric quantities such as crack opening displacements and crack tip opening angles, cannot be measured directly. In practice, even the above geometric parameters are difficult to quantify and are often computed by using analog models of the crack. Strain energy release rate and stress intensity factor in linear elastic fracture mechanics, which is a well established analog model of the crack, can be computed accurately by using modern numerical codes. The various fracture packages for these codes have been verified by a recent benchmark problem [25] and thus should provide correct numerical solutions to well-defined boundary value problems. Once the strain energy release rate or stress intensity factor is determined, the onset of brittle fracture can be predicted if the critical values of these quantities are known. Their elastic-plastic extension, the J-integral, has also been used with some success in predicting the onset of ductile fracture. Laws governing other fracture phenomena, such as stable crack growth under large scale yielding, are being investigated through empirical correlations of fracture data with computed fracture parameters.

An approach which has been used recently to establish a stable crack growth criterion is to input actual crack growth data as additional boundary values to an elastic-plastic finite element code. Kanninen et al. [26] used the finite element code in its "generation mode" to study stable crack growth

and instability of A533-B steel and 2219-T87 aluminum center-crack and compact specimens. A similar approach was used by Shih et al. [27] who studied stable crack growth and instability of A533-B compact specimens. Experimentally determined load-line displacement versus crack length relation, as shown in Figure 8, was used to simulate crack extension in the two-dimension finite element model shown in Figure 9. Two sets of elastic-plastic analyses based on $J_2$ deformation and $J_2$ flow theories of plasticity were conducted. Figure 10 shows excellent agreements between the measured and computed applied load versus load-line displacement relations obtained by these two numerical analyses. The computed fracture parameters included the crack opening displacement (COD), the crack opening angle (COA), the J-integral and the rate of change of J-integral, dJ/da. Since the fracture criterion for stable crack growth must be independent of specimen geometry and crack extension, these fracture parameters were then scrutinized for constancy during crack extension. Typical dJ/da and COA variations with crack extensions obtained by Shih et al. are shown in Figure 11 and 12, respectively. Both Kaninnen and Shih concluded from their hybrid experimental-numerical investigations that the COA was an attractive fracture criterion for stable crack growth in the presence of large scale yielding.

The above studies demonstrate the utility of the hybrid experimental-numerical technique in extracting candidate fracture parameters which cannot be obtained directly from either the experimental or the numerical analysis alone. The hybrid experimental-numerical technique provided computed fracture parameters, such as J and dJ/da, under actual test conditions and not under assumed test conditions which normally would have been prescribed in pure numerical analysis. The technique also yielded numerically consistent COD and COA which in theory are measurable but in practice are difficult to determine.

The elastic-plastic finite element codes with fracture packages, including those mentioned above, are yet to be subjected to the rigorous scrutiny leveled on the elastic codes. The wide variations in the J-integrals of the mid-1970's [28] hopefully have been reduced if not eliminated in the elasto-plastic finite element codes of today.

## DYNAMIC FRACTURE

The state of science on dynamic fracture mechanics studied with dynamic photoelasticity has been presented by J. W. Dally in his 1979 William M. Murray Lecture [29]. He noted that the crack tip state of stress provided by dynamic photoelasticity and dynamic caustic* techniques have and will continue to enhance our understanding on the complex phenomena of dynamic crack propagation. Dynamic fracture studies by these techniques, however, are limited to photoelastic polymers and to plane stress problems when photoelastic coatings or caustics are used. The hybrid experimental-numerical technique, when used with the generation mode of finite element or finite difference method will extract dynamic fracture parameters in opaque materials as well as in non-plane stress problems. These dynamic codes which, unlike the well-studied static codes, required verificaion prior to its used in dynamic fracture mechanics. Fracture dynamic results generated by various two-dimensional elasto-dynamic finite difference codes [30,31] and finite element codes [32,33] have been compared with dynamic caustic results of fracturing polymeric specimens [34,35]. Similar verification studies have been conducted with dynamic photoelasticity [36].

The verified numerical code can also be used to check ancillary results

*Added by the author.

deduced from the original experimental results, such as the variation in input work, which cannot be easily measured, during the fracture process. Numerical analysis also provides the transient energy partition for the input boundary and initial conditions. Such energy partition can then be used to check the hypothesis used in deducing the experimentally determined energy partition. The legend of Figure 13 shows an internally notched, semicircular photoelastic specimen which was loaded with end rotation and shear deformation [37]. The reported dynamic fracture toughness versus crack velocity relation [29] was used as a dyanmic fracture criterion to execute a dynamic finite element code in its application mode which yielded the crack propagation and dynamic stress intensity factor histories [38] which are in good agreement with the numerical results. Having verified the numerical modeling of the photoelastic experiment, the energies during crack propagation were computed and plotted as shown in Figure 14. The internal consistency in the computed energy partition verifies the basic postulates of negligible viscoelastic damping and negligible energy dissipation at the finite specimen boundaries during the dynamic crack propagation period.

A relatively simple application of the hybrid technique is the determination of the dynamic stress intensity factor in an impacted notch bend specimen. Measured time variations in the striker load were input to the finite element model of a dynamic finite element code which was then used to compute the time variations in the dynamic stress intensity factor [40]. The numerical code was also verified by comparing the computed and measured dynamic strains near the crack tip as shown in Figure 15. Figure 16 shows the variations in dynamic and the corresponding static stress intensity factors with time prior to the crack propagation. These results show the inadequacy of the static stress intensity factor which was computed by using a static formula

and the instantaneous striker load. It also indicates the futility in interpreting such impact fracture response without the use of proper dynamic analysis [41]. As a verification of codes, Figure 17 shows the agreement between three independent dynamic fracture analyses of another impacted three point bend specimen [42].

Figure 18 shows a wedge loaded, modified-tapered double cantilever beam (WL-MTDCB) specimen which was fabricated from plate glass. The specimen was 25% side-grooved to guide the propagating crack. The flexible, long tapered beam sections was designed to lessen the friction with the silicon carbide loading pin. The specimen was wedge-loaded to fracture in a 500-kg Instron testing machine and the crack extension history was recorded by a KRAK-GAGE and associated instrumentation [43]. Figure 19 shows typical crack length versus time data which is characterized by the unambiguous initial period of crack acceleration and which has not been observed in dynamic fracture of metals and photoelastic polymers. The average of the two data sets, which is represented by a solid curve in Figure 19, was used to drive a dynamic finite element code in its generation mode. The resultant $K_I^{dyn}$ as well as the static stress intensity factor, $K_I^{stat}$, which was also computed by finite element analysis, are shown in Figure 20. Although it is not obvious from Figure 20, unlike the dynamic fracture of metals and polymers, the crack never arrested in these and other ceramics WL-MTDCB specimens [40]. Thus the $K_I^{dyn}$ versus a curve in Figure 21 should continue past the nominal static fracture toughness $K_{IC}$ = 0.73 Mpa m as indicated by the dashed lines. Notable is the lack of the typical gamma-shaped $K_I^{dyn}$ versus a commonly observe in metals and polymers.

## CONCLUSION

The hybrid experimental-numerical technique yields reliable information which cannot be obtained by the single use of either the experimental or numerical technique. The utility of the hybrid experimental-numerical technique in experimental mechanics is demonstrated by case studies in two- and three-dimensional stress analysis, biomechanics and fracture mechanics.

## ACKNOWLEDGEMENTS

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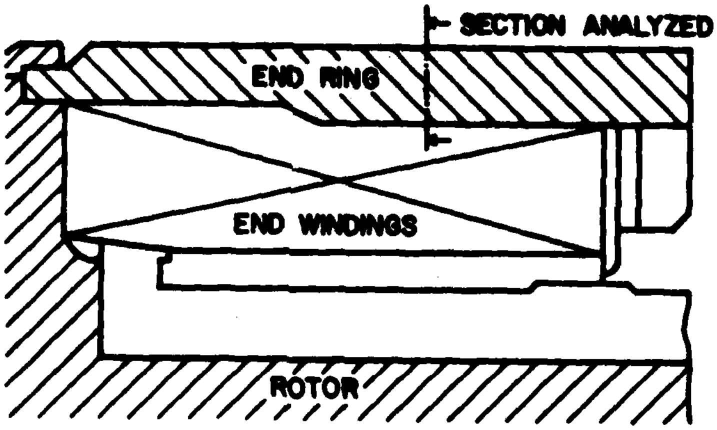



Fig. 1a End Ring Assembly on Rotor

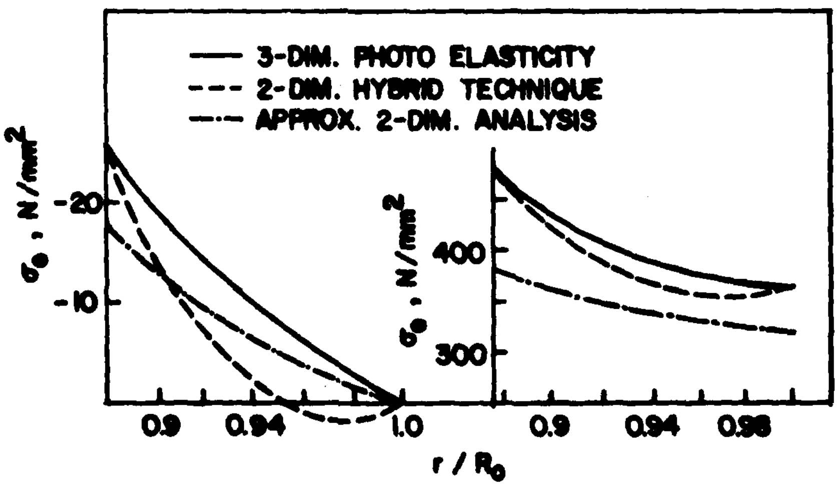



Fig. 1b Hoop Stress in End Ring Due to
Centrifugal Forces

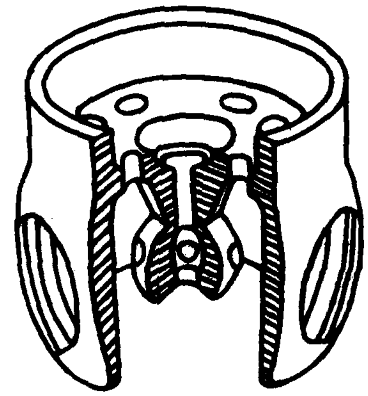


Fig. 2a The Model of the Working Wheel of a Water Turbine

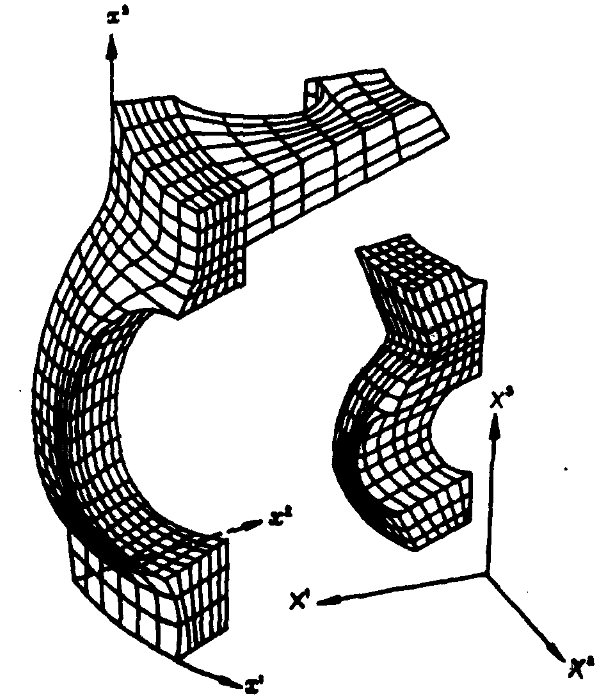
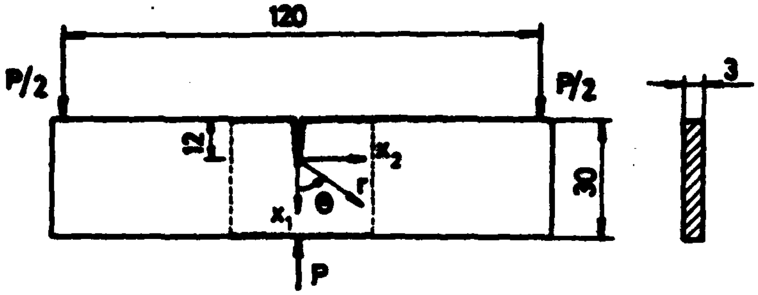



Fig. 2b Finite Difference Network



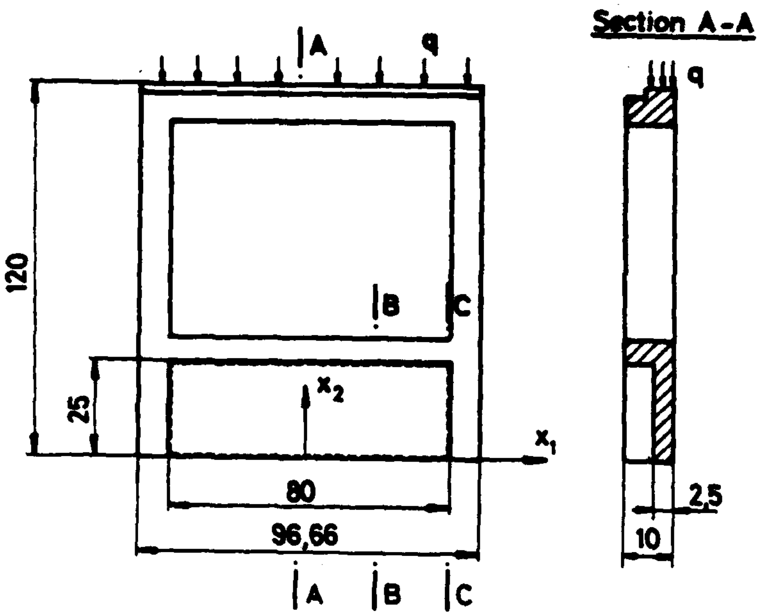




Fig. 3  Plate-Stiffened Frame

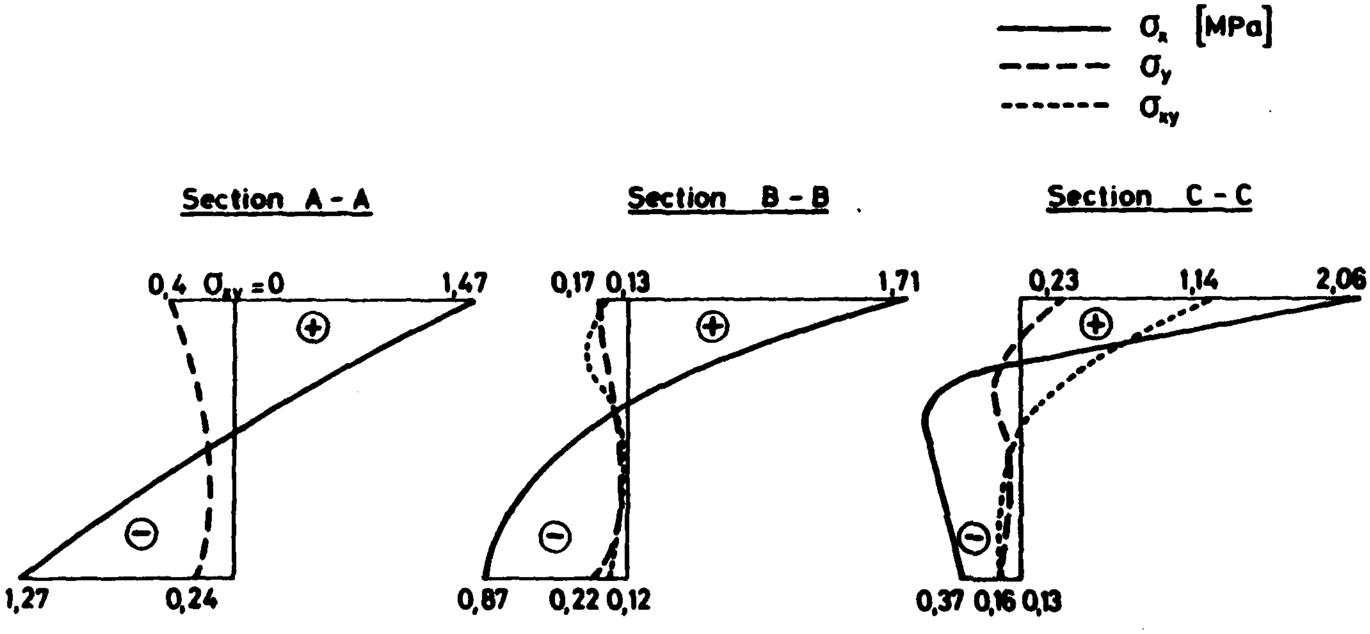



Fig. 4  Stresses in Plate Stiffened Frame

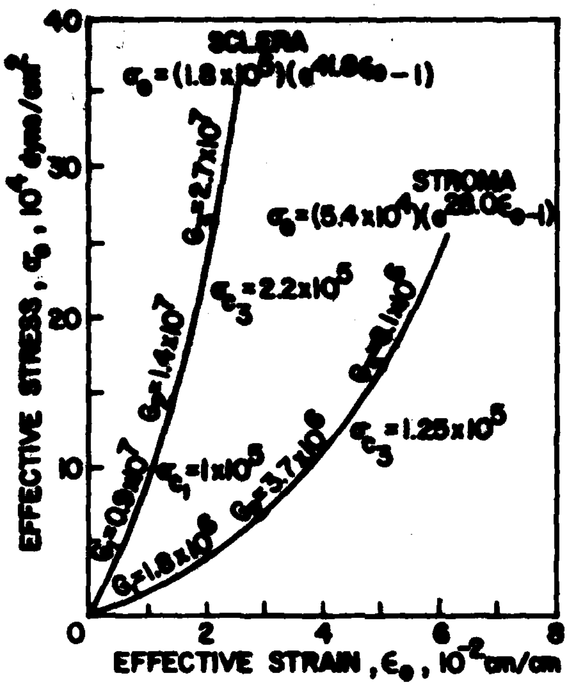



Fig. 5 Trilinear and Experimental Stress-Strain Relation

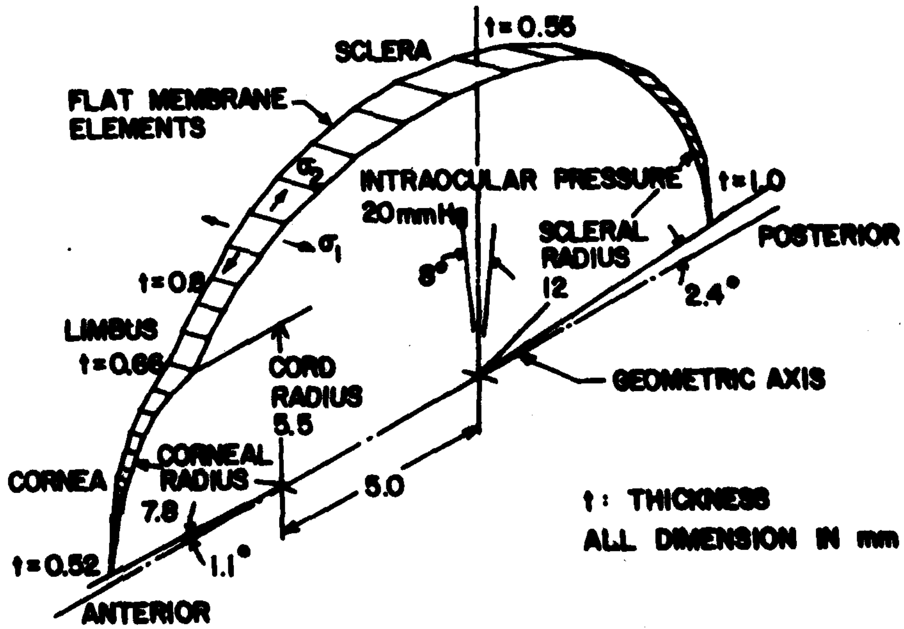



Fig. 6 Membrane Model Finite Element Arrangement

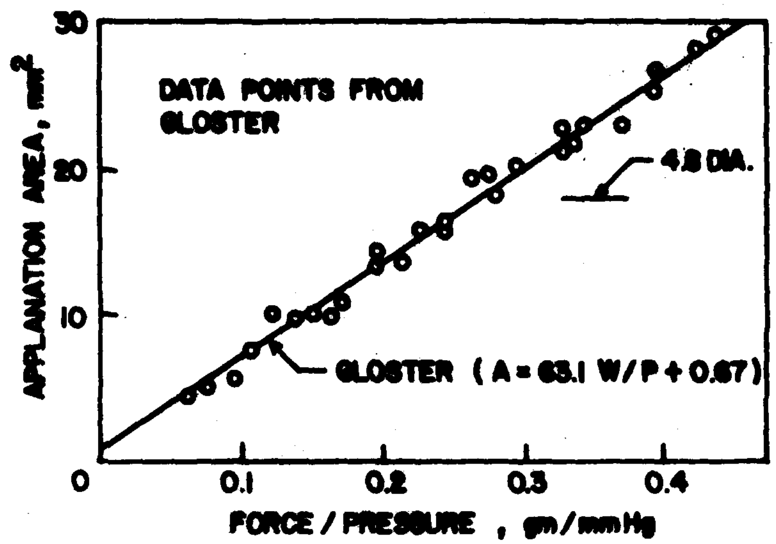



Fig. 7 Applanation Force/Intraocular Pressure vs Applanation Area

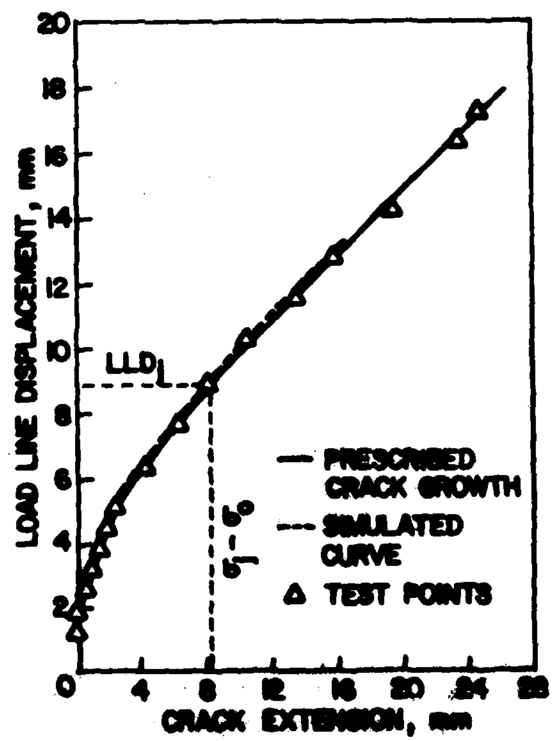



Fig. 8 Prescribed and Simulated Load-Line Displacement vs Crack Extension for 4T Compact Specimen

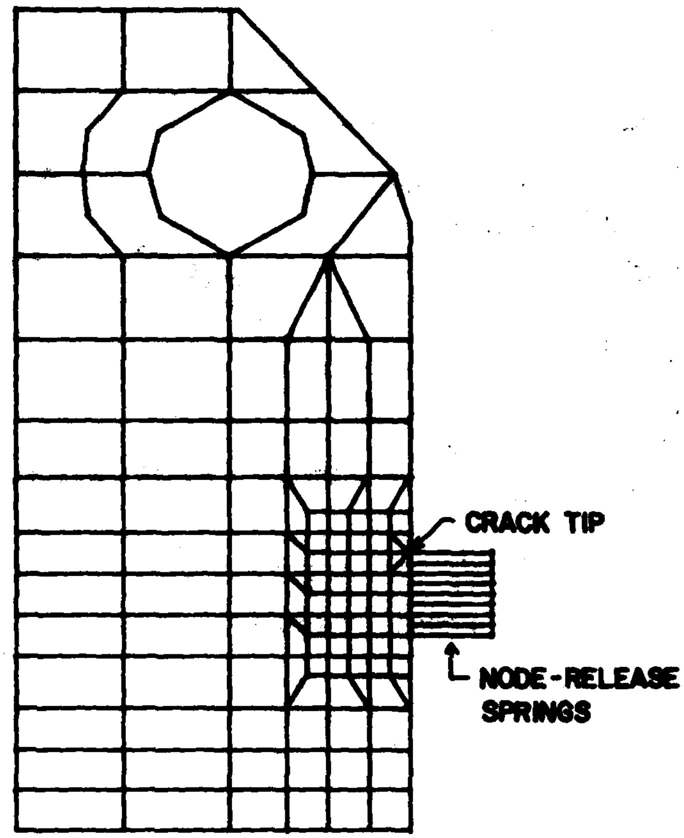



Fig. 9 Finite-Element Model for 4T Compact Specimen

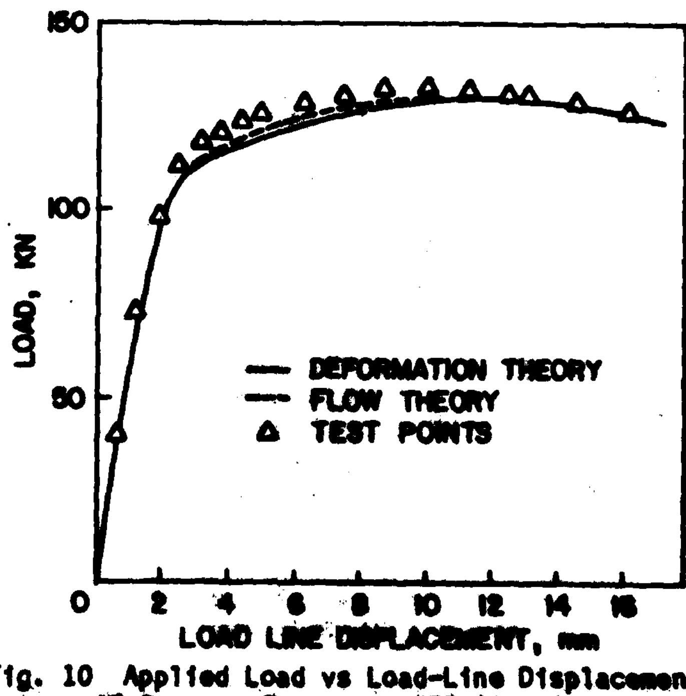



Fig. 10 Applied Load vs Load-Line Displacement for
4T Compact Specimen, 25% Side-Grooved,
$W-a_o$ = 40 mm (1.593 in.)

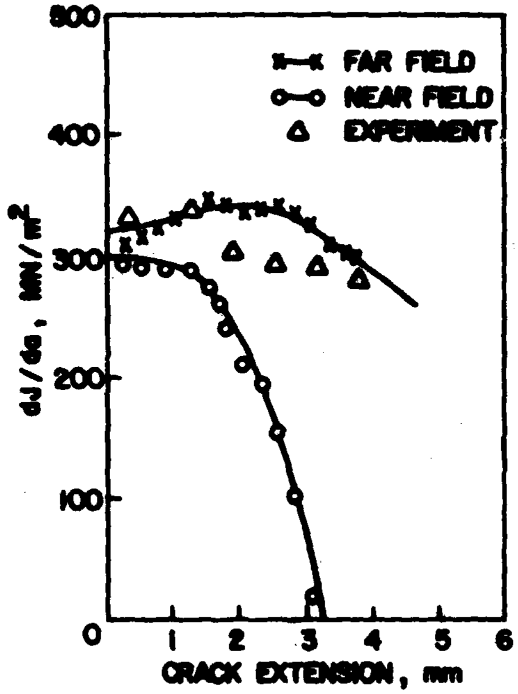



Fig. 11 dJ/da for Compact Specimen T-61, a/W = 0.801

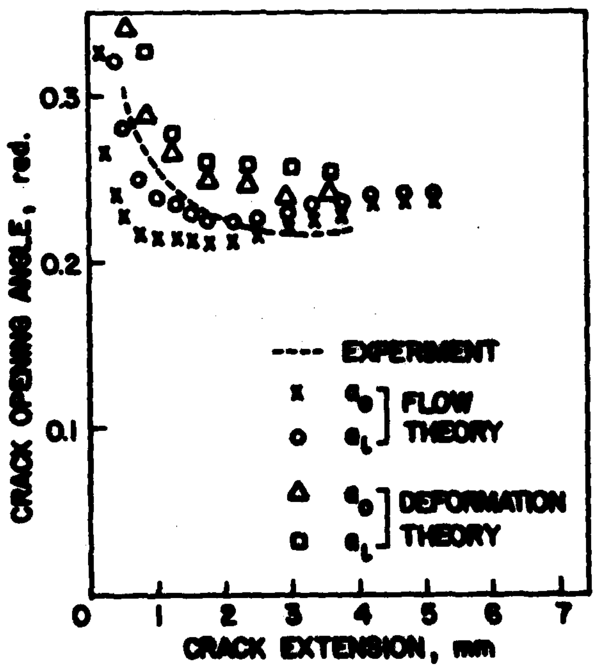



Fig. 12 Crack-Opening Angle vs Crack Extension for 4T Compact Specimen

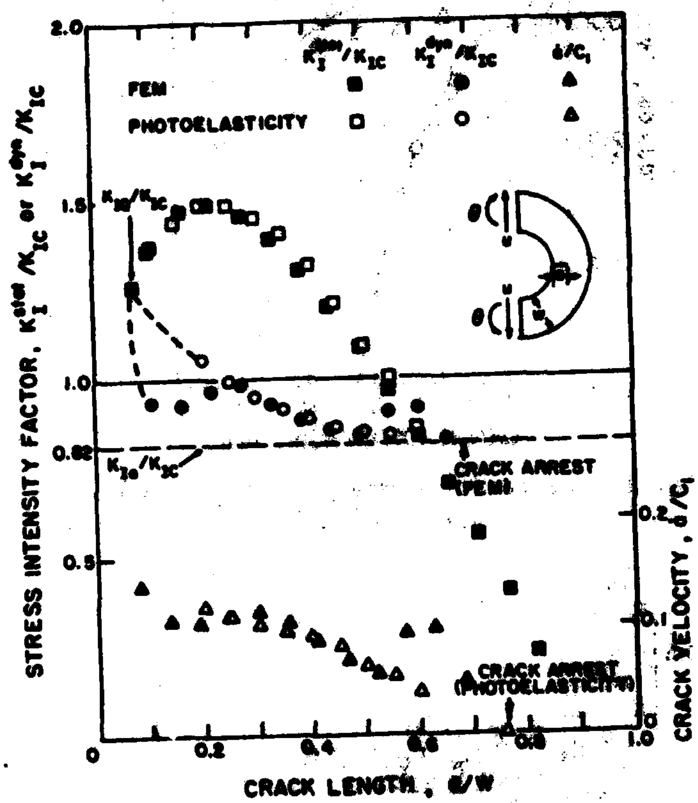



Fig. 13 Stress Intensity Factors and Crack Velocities of Internally Notched Semi-Circular Homalite-100 Specimen Subjected to End Rotation and Shear Formation

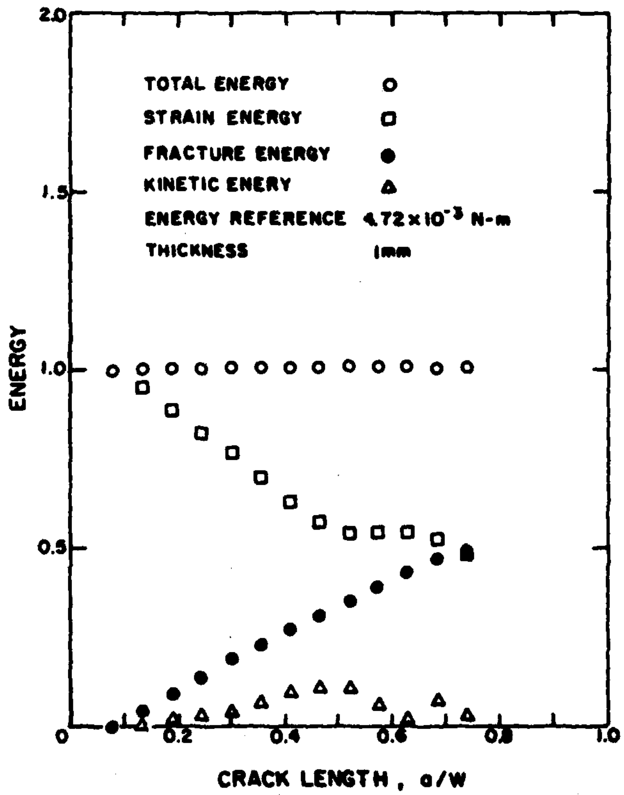



Fig. 14 Energies in Internally Notched, Semi-Circular Homalite-100 Specimen Subjected to End Rotation and Shear Deformation

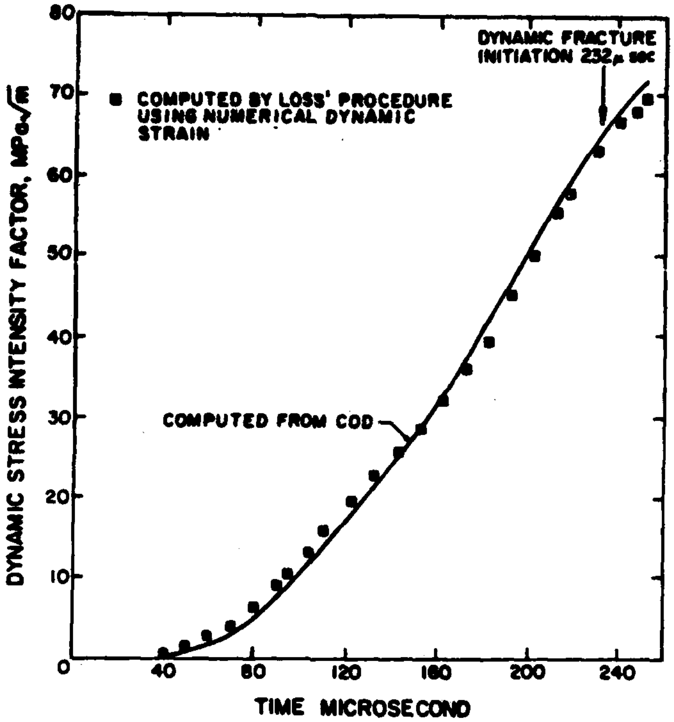



Fig. 15 Dynamic Strain at Location (1), A533B Bend Specimen

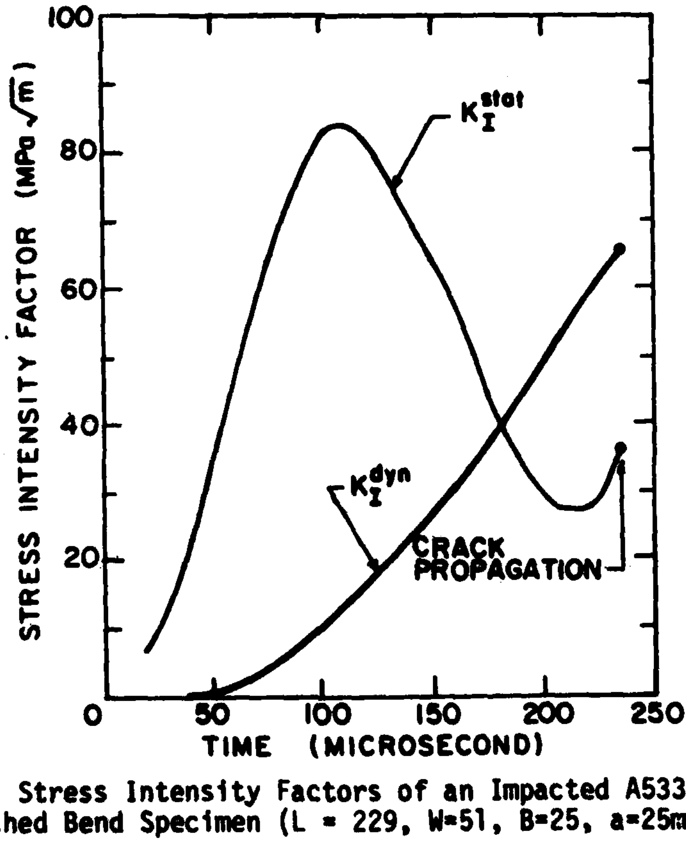



Fig. 16 Stress Intensity Factors of an Impacted A533B Steel Notched Bend Specimen (L = 229, W=51, B=25, a=25mm)

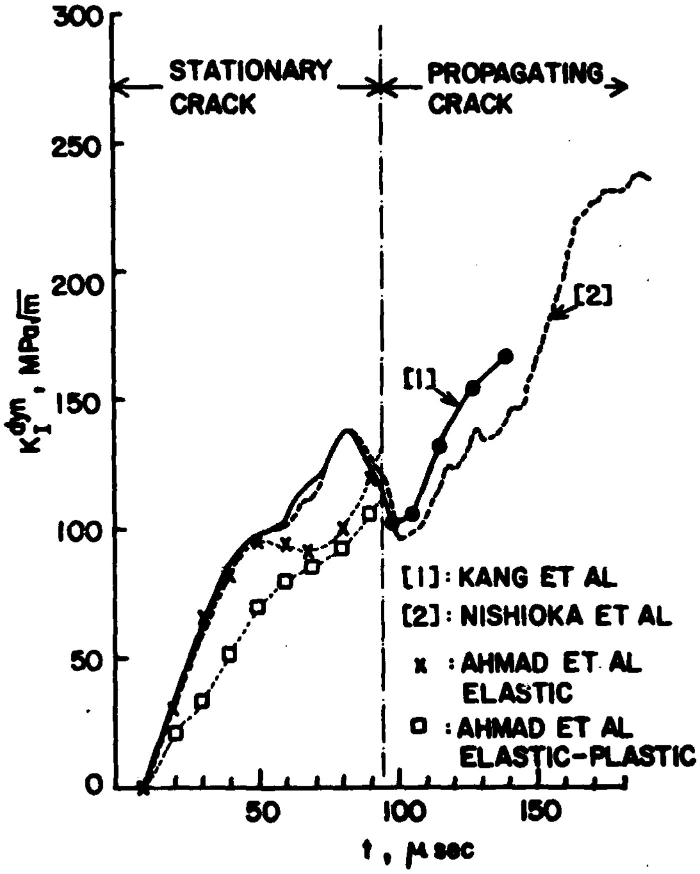



Fig. 17 Dynamic Stress Intensity Factor of a Dynamic Tear Test Specimen

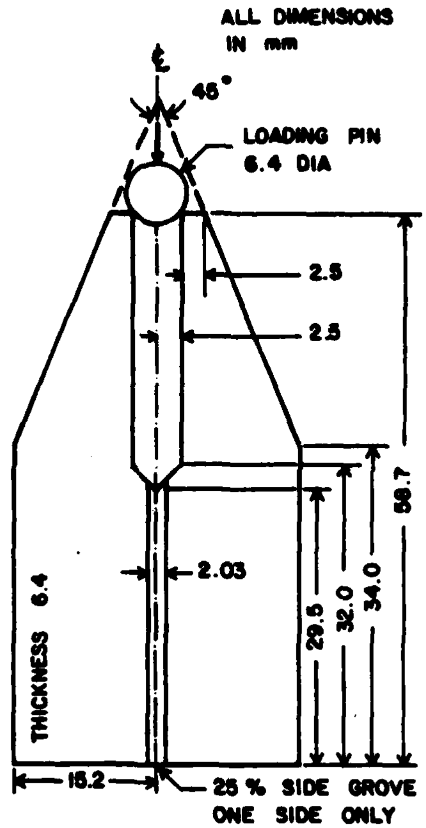



Fig. 18 Glass WL-MTDCB Specimen

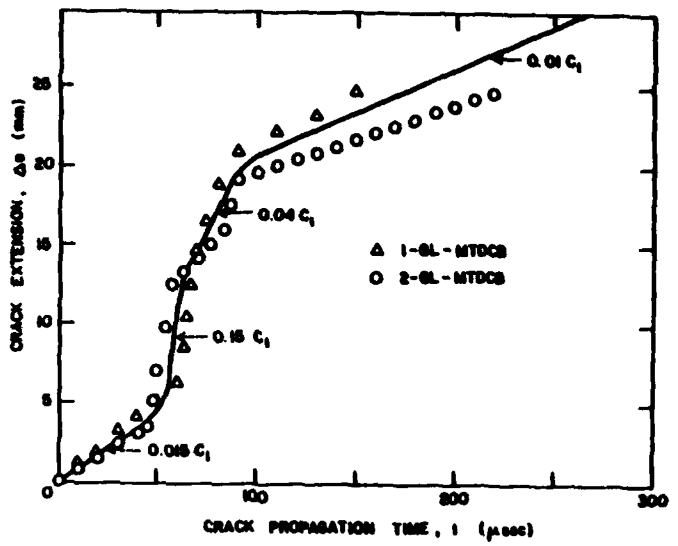



Fig. 19 Crack Extension vs Time of Fracturing WL-MTDCB Glass Specimen

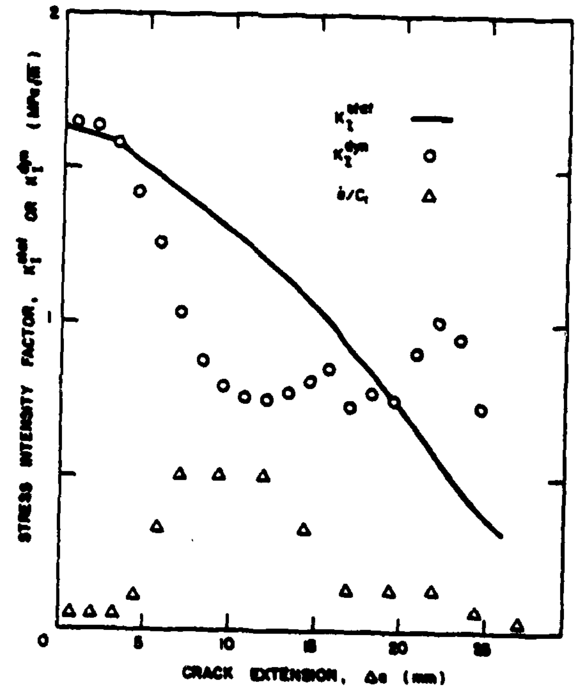



Fig. 20 Stress Intensity Factors and Crack Velocities in a Fracturing WL-MTDCB Glass Specimen

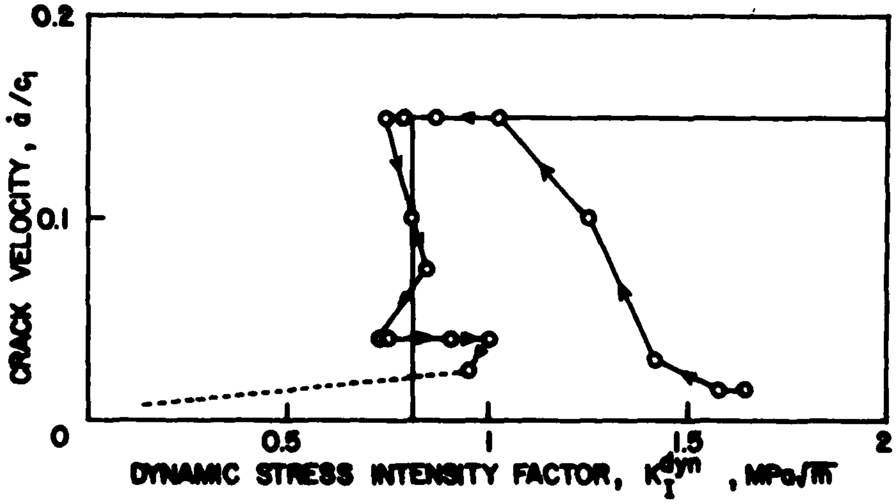



Fig. 21 Dynamic Fracture Toughness vs Crack Velocity Relation for Glass

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The hybrid experimental-numerical stress analysis technique, which saw limited applications during the 1950's, has been resurrected with the vastly improved numerical techniques of the 1970's. By inputing the experimental results as initial and boundary conditions, modern computer codes can be executed in its generation and application modes to yield results which are unobtainable when only one of the two techniques is used. The hybrid technique thus exemplifies the complementary role of the experimental and numerical techniques

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